JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 20, No. 4, July–August 1997

Geomagnetic Attitude Control of Satellites Using Generalized Multiple Scales

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The dynamics of single- and dual-spin satellites utilizing geomagnetic attitude control with specific control laws are analyzed. Stability criteria and approximate descriptions of the motion of the satellite are obtained in an analytical parametric form using the generalized multiple scales method. By means of this method, the rapid and slow parts of the dynamics are systematically separated, leading to insight into the nature of the system. Finally, a good agreement between the analytical approximations and numerical solutions is demonstrated.

I. Introduction

THE techniques of controlling the attitude of a satellite by using the interaction between the satellite and the environmental magnetic field are attractive because they eliminate the need for the control propellant, which usually limits the lifetime of a satellite. The control torque can be generated by passing electric currents through a coil; the currents then produce a magnetic dipole. The desired control torque is then generated by the interaction of this magnetic dipole with the geomagnetic field. Such a simple generation of control torques results in enhanced system reliability.

The major challenges in the magnetic attitude control for satellites are the development of the control law and the prediction of the performance of the system under the specific control law. Even though the concept of the magnetic attitude control itself is relatively simple, the resulting dynamic equations of the satellite system utilizing this type of control are quite complicated due to the varying magnitude of the geomagnetic field experienced by the satellite from time to time in its orbit. In general, this necessitates the analysis of the system equations with time-varying coefficients.

Several investigations on this subject have been reported.[1–8] In Refs. 1–5, single-spin axisymmetric satellites are considered. Renard[1] and Lindorfer and Muhlfelder[2] described the open-loop magnetic attitude control laws, which are commanded from the ground. Wheeler[3] developed a closed-loop magnetic control law for active nutation damping of a spinning axisymmetric satellite. Sorensen[4] and Shigehara[5] also dealt with feedback magnetic control systems of an axisymmetric spinning satellite. In Ref. 4 a minimum energy control law is developed, whereas in Ref. 5 switching control is used. The dual-spin satellite cases are reported in Refs. 6–8. Goel and Rajaram[6] investigated a magnetic control system for a dual-spin satellite in a near-equatorial orbit, where the geomagnetic field variation is neglected. The linearized governing equations are time invariant and, hence, standard control theory can be applied. Alfriend[7] studied the geomagnetic control system for a dual-spin satellite in an orbit at any inclination. He applied the multiple time scales (MTS) method (with linear scales) for his analysis and was able to obtain a stability criterion for a specific control law. His analysis, however, was based on the assumption that the Earth is nonrotating, so that some of the results are valid only for satellites in low-altitude orbits. Another more general work is by Stickler and Alfriend[8] in which the use of magnetic attitude control for initial acquisition and for on-orbit control of a dual-spin satellite is also discussed.

This paper describes a closed-loop magnetic attitude control system using a specific control law with single- and dual-spin satellite configurations. In the single-spin satellite case, this work differs from the previous ones by the use of a different control law with a general asymmetric satellite spun about an axis of maximum or minimum moment of inertia. Thus, the results obtained are more general and subsume the axisymmetric satellite as a special case. In the dual-spin case, besides the difference in the control law, we also include the Earth rotation effect. Hence, the results obtained are not limited to low-orbit satellites only but can also be applied to medium- or high-altitude satellites, depending on the validity of the Earth's magnetic model.

In general, the difficulties in analyzing the properties of the magnetic attitude control system arise from the fact that no relatively easy methods are available for analyzing a time-varying system. The MTS method used by Alfriend[7] to study the geomagnetic control system of a dual-spin satellite is promising because it provides not only the approximate solutions of a problem but also physical insight. However, only linear scales are used in the MTS approach. The MTS approach has been generalized by Ramnath et al.[9] and Ramnath and Sandri[10] in the generalized multiple scales (GMS) method to also include nonlinear and complex scales. The GMS method is used in the present analysis.

II. Control Law

The purpose of the magnetic attitude control considered is to maintain the pitch axis of the satellite in its nominal direction, normal to the satellite's orbital plane. The magnetic torque is produced by the interaction between the geomagnetic field and the onboard magnet. We assume that the magnet used for attitude control is a magnetic dipole $\boldsymbol{M_c}$ aligned with the pitch axis of the satellite. The strength of the magnetic dipole is determined by the control law

$$\boldsymbol{M_c} = [K_1 B_\chi \chi - K_2(B_\psi \dot{\chi} - B_\chi \dot{\psi})]\,\boldsymbol{i}_\theta \equiv M_c\,\boldsymbol{i}_\theta \tag{1}$$

where K_1 and K_2 are constant control gains and χ and ψ are perturbation angles around the x and y axes of the satellite, respectively, as defined later. B_χ and B_ψ are the components of the geomagnetic field vector in the x and y axes, respectively, and $\boldsymbol{i}_\theta$ is a unit vector in the direction of the pitch axis of the satellite. This control law requires some angular and angular rate sensors and also some magnetosensors to measure the magnitude of some components of the geomagnetic field.

This form of the control law is used in both the single- and dual-spin satellites considered. Note, however, that although the control laws used in the two cases appear to be similar, they are actually different, inasmuch as the definitions of body-fixed axes are slightly different, as will be described later.

We will first consider spinning asymmetric satellites.

III. Spinning Asymmetric Satellites

A. Equations of Motion

The coordinate systems employed in the analysis are described next. The Earth is assumed to have a fixed position in space. $X_I Y_I Z_I$ is the geocentric inertial coordinate system. The X_I and Y_I axes lie in the equatorial plane, and the Z_I axis is aligned with the Earth's polar axis. The orbiting coordinate system $X_o Y_o Z_o$ has its origin at

Received Nov. 7, 1994; revision received Jan. 7, 1997; accepted for publication March 26, 1997.

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the center of mass of the satellite. The X_o axis lies on the satellite's orbital plane and points radially outward. The Y_o axis also lies in the satellite's orbital plane and is in the direction of the motion of the satellite's center of mass, perpendicular to the X_o axis. The Z_o axis completes the right-handed Cartesian coordinate system and, thus, is normal to the orbital plane. Note that the orientation of this coordinate system is independent of the orientation of the satellite. The body-fixed coordinate system $X_bY_bZ_b$ also has its origin at the center of mass of the satellite, and the axes X_b, Y_b, and Z_b coincide with the satellite principal axes. We will denote the principal moments of inertia of the satellite about X_b, Y_b, and Z_b as I_x, I_y, and I_z respectively. Because the pitch motion is assumed to be nominal, the χ and ψ angles describe the perturbations with respect to the nominal position of the body-fixed coordinate system.

The orbit of the satellite is assumed to be circular, and thus the orbital angular speed Ω is constant. The satellite is also considered to be rigid and may have no axis of symmetry. The spin axis of the satellite is the Z_b axis, and in the nominal condition this axis is normal to the satellite's orbital plane. We assume that the spin axis is the principal axis of maximum or minimum moment of inertia. Also the spin rate ω_s of the satellite is assumed to be constant. This assumption also implies that the pitch motion of the satellite is under control such that its nominal motion can always be attained. The geomagnetic field is assumed to be represented by a tilted magnetic dipole passing through the center of the Earth. This magnetic dipole is fixed to the Earth at an angle γ ($\approx$11.4 deg) with respect to the Earth's polar axis. With this model, the geomagnetic field vector is given by[11]

$$\boldsymbol{B} = \left(\mu_b / R^5\right)\left[R^2\, \boldsymbol{i_m} - 3(\boldsymbol{i_m}\cdot\boldsymbol{R})\boldsymbol{R}\right] \tag{2}$$

where $\mu_b = 7.943 \times 10^{15}$ weber·m is the geomagnetic dipole moment, $\boldsymbol{R}$ is the radius vector with its magnitude R, and $\boldsymbol{i_m}$ is the unit vector in the opposite direction to the dipole moment vector.

The magnetic control torque exerted on the satellite is $\boldsymbol{L_c} = \boldsymbol{M_c} \times \boldsymbol{B_b}$, where $\boldsymbol{M_c}$ is given by the control law (1) and $\boldsymbol{B_b} \equiv [B_\chi \;\; B_\psi \;\; B_\theta]^T$ is the geomagnetic field vector expressed in the body-fixed axes. By including this control torque and assuming small angles and angular rates, the equations of motion of the satellite are given by

$$\ddot{\chi} + (K_2/I_y)B_\psi^2\dot{\chi} + \left[r_1(\omega_s+\Omega)^2 - (K_1/I_y)B_\chi B_\psi\right]\chi$$
$$+ [(1-r_1)(\omega_s+\Omega) - (K_2/I_y)B_\chi B_\psi]\dot{\psi} = 0 \tag{3}$$

$$\ddot{\psi} + (K_2/I_x)B_\chi^2\dot{\psi} + r_2(\omega_s+\Omega)^2\psi - [(1-r_2)(\omega_s+\Omega)$$
$$+ (K_2/I_x)B_\chi B_\psi]\dot{\chi} + (K_1/I_x)B_\chi^2\chi = 0 \tag{4}$$

where

$$r_1 \equiv \frac{I_z - I_x}{I_y}, \qquad r_2 \equiv \frac{I_z - I_y}{I_x} \tag{5}$$

B_χ and B_ψ are quasiperiodic and depend on the angle of inclination i of the satellite orbit and the angle u between the line of the ascending node and the projection of the negative magnetic dipole vector onto the equatorial plane. Clearly, by the assumption that the spin axis is the axis of maximum or minimum moment of inertia, r_1 and r_2 are both positive or negative, respectively. Further, because the inertia properties of a body obey the triangle inequality, r_1 and r_2 both lie between 0 and 1. Other disturbance torques such as those due to gravity gradient are neglected in the present analysis.

Because the satellite's spin rate is normally much larger than the orbital angular speed, we have

$$\Omega/\omega_s = \epsilon, \qquad 0 < |\epsilon| \ll 1 \tag{6}$$

Without loss of generality we assume that at $t = 0$, the satellite is at the ascending node and $u = 0$, so that we can write

$$\Omega t = \epsilon\bar{t} \tag{7}$$

$$u = \omega_e t = n\epsilon\bar{t} \tag{8}$$

where $\bar{t} \equiv \omega_s t$ is a nondimensional time and $n \equiv \omega_e/\Omega$ is the ratio of the Earth's spin rate and the orbital angular speed of the satellite. This ratio is a small number for a low-orbit satellite and is 1 for a geosynchronous satellite. The magnitude of the geomagnetic field is relatively small. For example, at a 500-km altitude orbit, $B_o (=\mu_b/R^3)$ is of the order of 10^{-5} in SI units. Therefore, it is reasonable to define K_1^* and K_2^* as follows:

$$\frac{K_1 B_o^2}{\sqrt{I_x I_y}\,\omega_s^2} = \epsilon K_1^*, \qquad \frac{K_2 B_o^2}{\sqrt{I_x I_y}\,\omega_s} = \epsilon K_2^* \tag{9}$$

where K_1^* and K_2^* are assumed to be of $\mathcal{O}(1)$. Using this parameterization and by means of cross differentiation and elimination of terms, the equations of motion for χ can be written in the decoupled form

$$\chi^{(4)} + \epsilon P_1(\bar{t})\chi^{(3)} + P_2(\bar{t})\chi^{(2)} + \epsilon P_3(\bar{t})\chi^{(1)} + P_4(\bar{t})\chi = 0 \tag{10}$$

where

$$P_1(\bar{t}) = K_2^*\left\{(a+b)f_1(\epsilon\bar{t}) - \left(a - b + \frac{4b}{1-r_1}\right)\right.$$
$$\left.\times [f_2(\epsilon\bar{t})\cos 2\bar{t} - f_3(\epsilon\bar{t})\sin 2\bar{t}]\right\} + \mathcal{O}(\epsilon)$$
$$\equiv p_{11}(\bar{t}) + \mathcal{O}(\epsilon)$$

$$P_2(\bar{t}) = 1 + r_1 r_2 + \epsilon\left(2 + r_2 + 2r_1 r_2\right.$$
$$+ \left\{K_1^* b + K_2^*\left[a(1-r_1) - b(3+r_2) + \frac{b(4+r_2)}{1-r_1}\right]\right\}$$
$$\left.\times [f_2(\epsilon\bar{t})\sin 2\bar{t} + f_3(\epsilon\bar{t})\cos 2\bar{t}]\right) + \mathcal{O}(\epsilon^2)$$
$$\equiv p_{21}(\bar{t}) + \epsilon p_{22}(\bar{t}) + \mathcal{O}(\epsilon^2)$$

$$P_3(\bar{t}) = \left[K_2^*(ar_1 + br_2) - K_1^* a(1-r_1)\right]f_1(\epsilon\bar{t})$$
$$+ \left\{K_1^*[a(1-r_1) + 4b] - K_2^*\left[a(2-r_1) + b(2+r_2)\right.\right.$$
$$\left.\left. + \frac{2b(1 + r_1 - r_2 + r_1 r_2)}{1-r_1}\right]\right\}$$
$$\times [f_2(\epsilon\bar{t})\cos 2\bar{t} - f_3(\epsilon\bar{t})\sin 2\bar{t}] + \mathcal{O}(\epsilon)$$
$$\equiv p_{31}(\bar{t}) + \mathcal{O}(\epsilon)$$

$$P_4(\bar{t}) = r_1 r_2 + \epsilon\left(5r_1 r_2 + \left\{K_2^*\left[2ar_1 + \frac{br_1(4+r_2)}{1-r_1}\right]\right.\right.$$
$$\left. - K_1^*[2a(1-r_1) + b(4-r_2)]\right\}[f_2(\epsilon\bar{t})\sin 2\bar{t}$$
$$\left. + f_3(\epsilon\bar{t})\cos 2\bar{t}]\right) + \mathcal{O}(\epsilon^2)$$
$$\equiv p_{41}(\bar{t}) + \epsilon p_{42}(\bar{t}) + \mathcal{O}(\epsilon^2) \tag{11}$$

with $a = \sqrt{(I_x/I_y)}$ and $b = \sqrt{(I_y/I_x)}$. Here $(\;)^{(i)}$ denotes the ith derivative with respect to $\bar{t}$. In the preceding equations, f_1, f_2, and f_3 are quasiperiodic functions with a period approaching that of the satellite's orbit.

The coefficients of this equation vary quasiperiodically in time, and their frequencies are a mixture of relatively high (due to satellite's spin) and low frequencies (due to orbital motion). One can also observe that some of the coefficients are of $\mathcal{O}(1)$ and some are small [of $\mathcal{O}(\epsilon)$]. The exact solutions of Eq. (10) cannot be obtained in general. A simplistic approach by neglecting the terms with small coefficients will lead to the equations of free rigid-body motion; hence, the result is not very useful. An asymptotic approach using the GMS method is used for the dynamic analysis.

B. Dynamic Analysis Using the GMS Method

The GMS method, developed in Refs. 9 and 10, is an asymptotic method designed to obtain approximate solutions of a dynamic problem in limiting cases. The approach is based on the concept of extension, by which the independent variable, time, is extended into a set of new independent timelike variables, which are called time scales. Each time scale captures a specific behavior of the system (fast or slow). The difference between the GMS and the MTS method is that in GMS the time scales can be nonlinear (corresponding to accelerating clocks) and/or complex, whereas in MTS the time scales are linear. Therefore, the results from the GMS approach, in general, are better than the ones from the MTS because the GMS approach can employ the natural scales of the system that may not be linear. For a more detailed discussion on GMS, see Refs. 9 and 10.

Two time scales are used in the present analysis, and as per Refs. 9 and 10, the independent and dependent variables are extended as follows: $\bar{t} \to \{\upsilon_0, \upsilon_1\}$ and $\chi(\bar{t}; \epsilon) \to \chi(\upsilon_0, \upsilon_1; \epsilon)$, where $\upsilon_0 = \bar{t}$ and $\upsilon_1 = \epsilon \int k(\bar{t})\, d\bar{t}$. The clock function $k(\bar{t})$ is determined in the course of the analysis. By this extension, the first two dominant equations obtained are

$$\mathcal{O}(\epsilon^0){:}\ \frac{\partial^4 \chi}{\partial \upsilon_0^4} + (1 + r_1 r_2)\frac{\partial^2 \chi}{\partial \upsilon_0^2} + r_1 r_2 \chi = 0 \tag{12}$$

$$\begin{aligned}\mathcal{O}(\epsilon){:}\ & k'''\frac{\partial \chi}{\partial \upsilon_1} + 3k''\frac{\partial^2 \chi}{\partial \upsilon_0 \partial \upsilon_1} + p_{21}k'\frac{\partial \chi}{\partial \upsilon_1} + 5k'\frac{\partial^3 \chi}{\partial \upsilon_0^2 \partial \upsilon_1} \\ & + 2p_{21}k\frac{\partial^2 \chi}{\partial \upsilon_0 \partial \upsilon_1} + 4k\frac{\partial^4 \chi}{\partial \upsilon_0^3 \partial \upsilon_1} + p_{11}(\upsilon_0)\frac{\partial^3 \chi}{\partial \upsilon_0^3} \\ & + p_{31}(\upsilon_0)\frac{\partial \chi}{\partial \upsilon_0} + p_{22}(\upsilon_0)\frac{\partial^2 \chi}{\partial \upsilon_0^2} + p_{42}(\upsilon_0)\chi = 0\end{aligned} \tag{13}$$

The dominant-order equation [Eq. (12)] yields

$$\chi(\upsilon_0, \upsilon_1) = A_1(\upsilon_1)e^{j\upsilon_0} + A_2(\upsilon_1)e^{j\sqrt{r_1 r_2}\,\upsilon_0} + cc \tag{14}$$

where $j = \sqrt{(-1)}$ and cc denotes the complex conjugates of the preceding terms. In this case, the dominant motion consists of two oscillatory modes with the dominant frequencies of 1 and $\sqrt{(r_1 r_2)}$. These frequencies correspond to the approximate natural frequencies of the torque-free rigid-body motion. In fact, if A_1 and A_2 are constant, Eq. (14) becomes the $\mathcal{O}(\epsilon)$ approximation of the torque-free rigid-body motion. Thus, the magnetic attitude control system will mainly influence the amplitude and only slightly alter the frequency of the motion. The amplitude and frequency corrections due to the magnetic control come from the subdominant-order analysis. For convenience, the two independent solutions of Eq. (14) are referred to as the first mode and the second mode, respectively. Note that the maximum limiting value of $r_1 r_2$ is 1, so that, in general, the second mode has lower frequency than the first mode. In the following, the first and second modes are studied separately.

1. First Mode

By substituting the first mode into the subdominant-order equation [Eq. (13)] and then separating the variables, the following pair of amplitude and clock equations are obtained:

$$A_1(\upsilon_1) = \bar{A}_{1_0}e^{-\upsilon_1}, \qquad \bar{A}_{1_0} = \text{arbitrary const} \tag{15}$$

$$\begin{aligned} & k_1''' - (4 - r_1 r_2)k_1' + j[3k_1'' - 2(1 - r_1 r_2)k_1] \\ &= M_1 + M_2(\epsilon\upsilon_0)\cos 2\upsilon_0 + M_3(\epsilon\upsilon_0)\sin 2\upsilon_0 \\ &\quad + j[N_1 + N_2(\epsilon\upsilon_0)\cos 2\upsilon_0 - N_3(\epsilon\upsilon_0)\sin 2\upsilon_0 + N_4(\epsilon\upsilon_0)]\end{aligned} \tag{16}$$

where

$$\begin{aligned} M_1 &= 3r_1 r_2 - r_2 - 2 \\ N_1 &= -\tfrac{5}{2}\left[a(1 - r_1)K_1^* - [a(1 - r_1) + b(1 - r_2)]K_2^*\right]U \end{aligned} \tag{17}$$

with

$$U = \tfrac{1}{4}\sin^2\gamma(1 + \cos^2 i) + \tfrac{1}{2}\cos^2\gamma\sin^2 i \tag{18}$$

It is clear that U is always positive and is constant for a particular orbit. Note that M_1 and N_1 are constant, whereas M_2, M_3, N_2, N_3, and N_4 are slowly quasiperiodic functions.

In most applications, only the particular solution of $k(\upsilon_0)$ is important because we always have the freedom to take the coefficients of the homogeneous solution to be zero. However, the possibility of resonance has to be examined. By resonance we mean that the inhomogeneous terms (terms in the right-hand side of the differential equation) contain the same type of function as the homogeneous solution so that secular, i.e., asymptotically nonuniform, terms are produced in the particular solution. If such terms exist, further analysis is needed. The homogeneous solution of Eq. (16) is

$$k_{h_1}(\upsilon_0) = \sum_{i=1}^{3} C_i e^{s_i \upsilon_0} \tag{19}$$

where C_i, $i = 1, 2, 3$, are arbitrary constants (which will be taken to be zero later) and s_i, $i = 1, 2, 3$, are the roots of the characteristic equation of the differential Eq. (16),

$$s^3 + 3js^2 - (4 - r_1 r_2)s - 2j(1 - r_1 r_2) = 0 \tag{20}$$

In general, the homogeneous solution is of the form

$$e^{\Re(s_i)\upsilon_0}[\cos\Im(s_i)\upsilon_0 + j\sin\Im(s_i)\upsilon_0]$$

with $\Re(s_i)$ and $\Im(s_i)$ denoting the real part and the imaginary part of s_i, respectively. Examination of Eq. (20) reveals that the resonance occurs only near the limiting case $r_1 r_2 = 1$. The GMS approximation is not valid near the resonance condition $[1 - r_1 r_2 = \mathcal{O}(\epsilon)]$ because the clock function will be of $\mathcal{O}(1/\epsilon)$ and, hence, the ordering of the extended equation of motion is rendered invalid. For the approximation to be accurate, $1 - r_1 r_2 \gg \mathcal{O}(\epsilon)$. Because the nutation frequency of the rigid body is $(\omega_s + \Omega)\sqrt{(r_1 r_2)}$, the preceding requirement means physically that the approximation for the first mode is accurate if the nutation frequency is significantly lower than the spin frequency.

By the earlier argument, it is possible to construct a stability criterion for the first mode without actually solving Eq. (16). From Eq. (15), it is clear that the stability of the first mode is determined by the constant real part of $k(\upsilon_0)$, which appears due to the contribution of N_1. The constant real part of $k(\upsilon_0)$ is

$$k_{r_1} = \frac{5}{2}\,\frac{a(1 - r_1)K_1^* + [a(1 - r_1) + b(1 - r_2)]K_2^*}{2(1 - r_1 r_2)}\,U \tag{21}$$

Because U is a positive constant and $0 < r_1 r_2 < 1$, then for asymptotic stability,

$$a(1 - r_1)K_1^* + [a(1 - r_1) + b(1 - r_2)]\,K_2^* > 0 \tag{22}$$

To predict the response of the first mode, the approximate solution for $k(\upsilon_0)$ will now be constructed. The slowly periodic factor $[M_i(\epsilon\upsilon_0)$ and $N_i(\epsilon\upsilon_0)]$ in the inhomogeneous terms, which is similar to $M_i(\epsilon\upsilon_0)\sin 2\upsilon_0$, will be treated as a constant in the integration step because for each cycle of the fast periodic factor the variation of the slowly periodic factor is very small. By doing so, we obtain the approximate response of the first mode as follows:

$$\begin{aligned}\chi_1(\bar{t}) = A_{1_0}\exp\left[\epsilon k_{r_1}\bar{t} + \epsilon\Re\left(\int k_1^*(\bar{t})\,d\bar{t}\right)\right] \\ \times \sin\left\{\bar{t} + \epsilon\Im\left[\int k_1^*(\bar{t})\,d\bar{t}\right] + \theta_{1_0}\right\}\end{aligned} \tag{23}$$

where A_{1_0} and θ_{1_0} are constants to be determined from the initial conditions and $k_1^*(\bar{t}) = k_1(\bar{t}) - k_{r_1}(\bar{t})$, which is periodic.

2. Second Mode

In a similar way, the pair of amplitude and clock equations obtained for the second mode are

$$A_2 = \bar{A}_{2_0}e^{-\upsilon_1}, \qquad \bar{A}_{2_0} = \text{arbitrary const} \tag{24}$$

$$k_2''' + (1 - 4r_1r_2)k_2' + j[3\sqrt{r_1r_2}k_2'' - 2\sqrt{r_1r_2}(1 - r_1r_2)k_2]$$
$$= V_1 + V_2(\epsilon\upsilon_0)\cos 2\upsilon_0 + V_3(\epsilon\upsilon_0)\sin 2\upsilon_0$$
$$+ j[W_1 + W_2(\epsilon\upsilon_0)\cos 2\upsilon_0 - W_3(\epsilon\upsilon_0)\sin 2\upsilon_0 + W_4(\epsilon\upsilon_0)] \quad (25)$$

where

$$V_1 = 3r_1r_2 - r_1r_2^2 - 2r_1^2r_2^2$$
$$W_1 = \tfrac{5}{2}\left\{-a(1 - r_1)K_1^* + [ar_1(1 - r_2) + br_2(1 - r_1)]K_2^*\right\}U \quad (26)$$

The homogeneous solution of Eq. (25) is of the same form as Eq. (19), where s_i are the roots of the characteristic equation

$$s^3 + j3\sqrt{r_1r_2}s^2 + (1 - 4r_1r_2)s + 2j\sqrt{r_1r_2}(1 - r_1r_2) = 0 \quad (27)$$

The resonance condition in this case is only possible for $r_1r_2 \approx 0$ and $r_1r_2 \approx 1$. As in the first mode, the GMS analysis is not accurate in the near-resonance condition, which is $r_1r_2 = \mathcal{O}(\epsilon)$ or $1 - r_1r_2 = \mathcal{O}(\epsilon)$ because, for this condition, the clock function $k_2(\bar{t})$ is of $\mathcal{O}(1/\epsilon)$. The approximation for the second mode is accurate for $r_1r_2 \gg \mathcal{O}(\epsilon)$ and $1 - r_1r_2 \gg \mathcal{O}(\epsilon)$. Physically, this means that the nutation frequency must be significantly higher than the orbital frequency but significantly lower than the spin frequency.

The right-hand side of Eq. (25) consists of constant and periodic terms. Hence, in the nonresonance condition, the particular solution of $k(\upsilon_0)$ will also consist of constant and periodic terms. From the expression for the amplitude of the second mode [Eq. (24)], we deduce that the stability of this mode is determined by the constant real part of the particular solution of $k(\upsilon_0)$,

$$k_{r_2} = \frac{5}{2}\frac{[ar_1(1 - r_2) + br_2(1 - r_1)]K_2^* - a(1 - r_1)K_1^*}{2(1 - r_1r_2)}U \quad (28)$$

For the second mode to be asymptotically stable, k_{r_2} must be positive. Because U is a positive constant, then the criterion for asymptotic stability of the second mode is as follows:

$$a(1 - r_1)K_1^* < [ar_1(1 - r_2) + br_2(1 - r_1)]K_2^* \quad (29)$$

This stability criterion together with the one for the first mode [Eq. (22)] must be satisfied to get an asymptotically stable roll motion. Note that these criteria lead to slightly different control conditions when the spin axis is along the axis of maximum or minimum moment of inertia.

Simplifying assumptions, as in the case of the first mode, are also used to obtain the approximate expression for $k(\upsilon_0)$. The resulting approximate second mode response is

$$\chi_2(\bar{t}) = A_{2_0}\exp\left[\epsilon k_{r_2}\bar{t} + \epsilon\Re\left(\int k_2^*(\bar{t})\,\mathrm{d}\bar{t}\right)\right]$$
$$\times \sin\left[\bar{t} + \epsilon\Im\left(\int k_2^*(\bar{t})\,\mathrm{d}\bar{t}\right) + \theta_{2_0}\right] \quad (30)$$

where A_{2_0} and θ_{2_0} are constants to be determined from the initial conditions and $k_2^*(\bar{t}) = k_2(\bar{t}) - k_{r_2}(\bar{t})$, which again is periodic.

The resulting motion is

$$\chi(\bar{t}) = \chi_1(\bar{t}) + \chi_2(\bar{t}) \quad (31)$$

with $\chi_1(\bar{t})$ and $\chi_2(\bar{t})$ given by Eqs. (23) and (30), respectively. Hence, the roll motion consists of two oscillatory modes and the amplitudes of both modes vary exponentially. Note that in general, the GMS solution subsumes the direct MTS solution. When k is a constant, GMS recovers the MTS case.

C. Summary and Performance Evaluation

The stability criteria for the attitude motion of a magnetically controlled asymmetric spinning satellite with the control law given by Eq. (1) have been derived. These criteria depend only on the inertia distribution, the spin rate, and the altitude of the satellite and do not depend on the other orbital parameters.

The time constant of the motion, on the other hand, depends not only on the inertia distribution and the spin rate but also on the orbital parameters of the satellite. From the preceding results, the approximate time constants of the body-roll/yaw motion are as follows.

First mode:

$$T_{c_1} = \frac{4(1 - r_1r_2)}{\epsilon 5\left\{a(1 - r_1)K_1^* + [a(1 - r_1) + b(1 - r_2)]K_2^*\right\}\omega_s U} \quad (32)$$

Second mode:

$$T_{c_2} = \frac{4(1 - r_1r_2)}{\epsilon 5\left\{[ar_1(1 - r_2) + br_2(1 - r_1)]K_2^* - a(1 - r_1)K_1^*\right\}\omega_s U} \quad (33)$$

These time constants can be useful to predict when the response reaches the steady-state condition. The normal steady-state criterion is that the response has stayed within 5% of its initial condition. As per common practice, the time to reach this steady-state condition is taken to be about three times the time constant of the motion ($t = 3T$).

In Eqs. (32) and (33), U is a constant for a particular orbit with a value depending on the inclination of the orbit. The value of U as a function of the inclination is plotted in Fig. 1. The value of U reaches a maximum when the satellite is in polar orbit and a minimum when the satellite is in equatorial orbit. Thus, the magnetic control system is more effective for satellites in an orbit at a high angle with respect to the equatorial plane.

The altitude of the satellite orbit also influences the effectiveness of the control system. The higher the altitude of the satellite, the smaller the magnitude of the geomagnetic field it encounters. Therefore, higher control gains are needed to achieve a certain time constant requirement.

The effectiveness of the magnetic control system also depends on the relative angle i of the geomagnetic dipole with respect to the

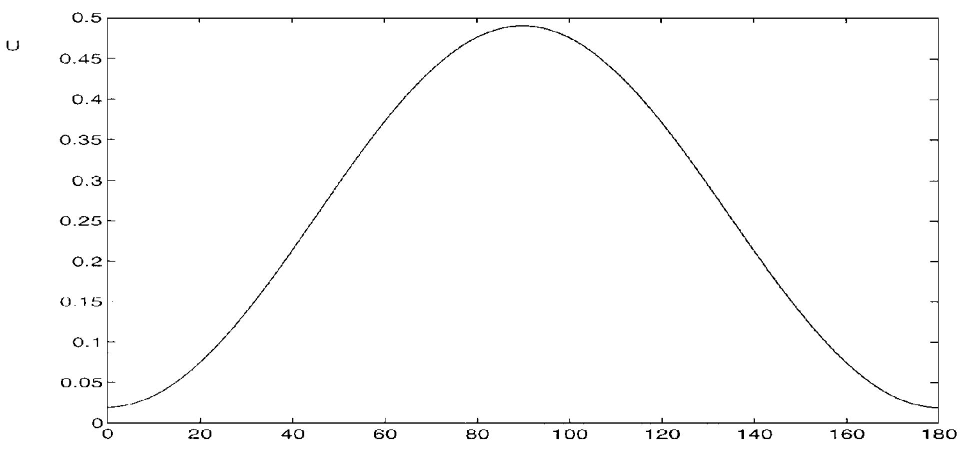

Fig. 1 Variation of U with inclination.

orbital plane of the satellite. The effectiveness (as indicated by U) will be a maximum when the geomagnetic dipole lies on the orbital plane because, in this case, for a certain magnitude of the onboard magnetic dipole, the torque produced is the largest. The approximate time constants must be slightly modified for this condition. However, for preliminary analysis purposes, the approximations (32) and (33) give practically useful and accurate predictions of the exact situation.

Spinning axisymmetric satellites, which are common in practice, are a special case of the satellites treated here. For this type of satellite, $r_1 = r_2 = r$ and $a = b = 1$, so that the stability criteria for the roll and yaw motion of the satellite become

$$K_1^* + 2K_2^* > 0 \tag{34}$$

$$K_1^* < 2rK_2^* \tag{35}$$

Further, the expressions for the approximate time constants of the roll/yaw motion for the first mode is simplified to be

$$T_{c_1} = \frac{4(1+r)}{\epsilon 5\left[K_1^* + K_2^*\right]\omega_s U} \tag{36}$$

and for the second mode

$$T_{c_2} = \frac{4(1+r)}{\epsilon 5\left[2rK_2^* - K_1^*\right]\omega_s U} \tag{37}$$

The earlier discussion on the performance of the system is also valid for this case.

IV. Comparison with Numerical Results

The explicit approximate solutions derived are now compared with the numerical solutions of the equations of motion for a satellite model with $I_x = 140$, $I_y = 80$, $I_z = 150$ kg · m^2, and spin rate $\omega_s = 0.2$ rad/s. The altitude of the orbit is 1000 km, and its inclination is 60 deg. For the numerical values used, $\epsilon = 0.005$. Note also that this satellite model does not violate the range of validity of the GMS results.

The stability criteria for the roll/yaw motion then become

$$0.6614K_1^* + 1.3229K_2^* > 0, \qquad 0.6614K_1^* < 0.6260K_2^* \tag{38}$$

Several numerical simulations were carried out near the stability boundary to examine the accuracy of the stability criteria. It was found that the stability criteria lead to an accurate stability prediction in all of the cases.

For response comparison purposes, we choose $K_1^* = 1$ and $K_2^* = 3$, which satisfy the stability criteria. Figure 2 shows the response of the satellite using numerical integration and the GMS method, presented on the long and short scales, for the same initial conditions in both simulations. The dominant frequencies predicted by using

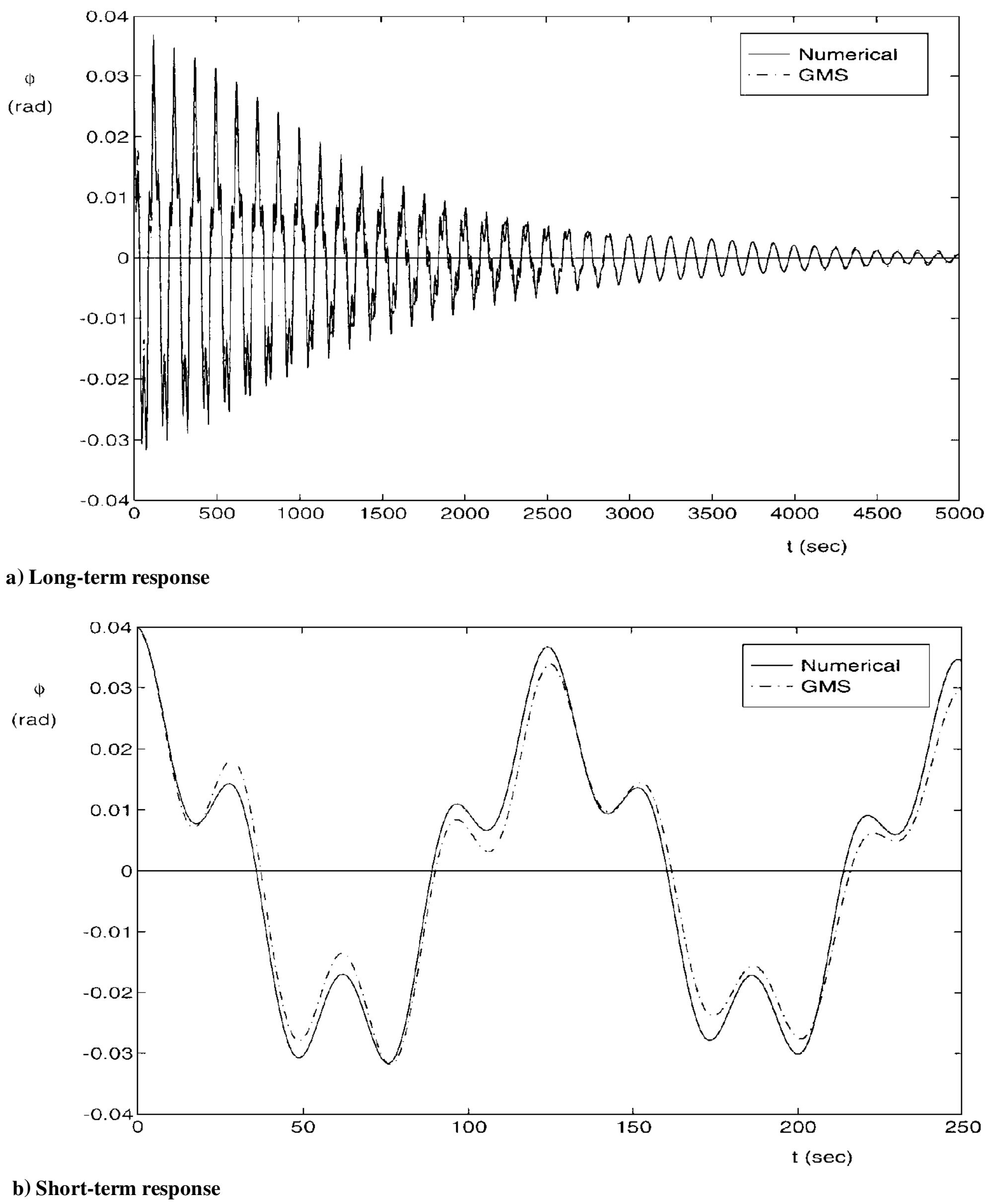

Fig. 2 Comparison of numerical and GMS results for roll motion.

the GMS technique are 0.2 and 0.0125 rad/s. We can see that these frequency predictions are accurate.

For the example satellite, the approximate time constants of the motion are $T_{c_1} = 610.14$ and $T_{c_2} = 1658.6$ s. The first and second modes, therefore, are expected to attenuate to within 5% of their initial values after 1830.42 and 4975.8 s, respectively. Indeed, it can be observed that the fast oscillation (first mode) attenuates significantly near 1900 s. Also the slow mode (second mode) becomes insignificant near 5000 s. Again, the approximation using the GMS approach is shown to be good.

We will now consider dual-spin satellites.

V. Dual-Spin Satellites

A. Equations of Motion

The inertial and orbiting coordinate systems for this case are the same as before. The body-fixed coordinate system is slightly different, however, because now there is a spinning part (rotor) and a nonspinning part (platform). The body-fixed coordinate system $X_bY_bZ_b$ has its origin at the center of mass of the satellite and is fixed to the satellite platform. The axes X_b, Y_b, and Z_b coincide with the satellite's principal axes and, as before, the principal moments of inertia of the satellite about X_b, Y_b, and Z_b are denoted by I_x, I_y, and I_z respectively. As per convention, the rotational motion about the X_b, Y_b, and Z_b axes are called yaw (ψ), roll (χ), and pitch (θ), respectively.

The platform may be asymmetric and is oriented such that one of its principal axes is in the direction of the radius vector from the center of the Earth to the satellite center of mass. The rotor of the satellite is axisymmetric, spinning about its axis of symmetry, which coincides with the pitch axis of the satellite and is normal to the orbit plane in the nominal condition and also coincides with one of the principal axes of the platform. Therefore, while spinning, the rotor does not change the inertia distribution of the satellite. The spin rate of the rotor is constant, and the orbit is assumed to be circular.

It is clear, from the earlier assumptions, that in the nominal condition the body-fixed coordinate system of the satellite coincides with the orbiting coordinate system ($\chi = \psi = \theta = 0$). For this reason, the actual attitude of the satellite is expressed in terms of deviations from the orbiting coordinate system. The purpose of the attitude control system is to maintain this nominal condition. We assume that the pitch motion is controlled separately and that we can always have $\theta = \dot{\theta} = 0$.

By using the control law Eq. (1) and by assuming small angles and angular rates, the satellite equations of motion are

$$I_y\ddot{\chi} + K_2B_\psi^2\dot{\chi} + \left[\Omega h - \Omega^2 I_x - K_1 B_\chi B_\psi\right]\chi - [h - \Omega(I_x + I_y) + K_2 B_\chi B_\psi]\dot{\psi} = 0 \tag{39}$$

$$I_x\ddot{\psi} + K_2B_\chi^2\dot{\psi} + \Omega[h - \Omega I_y]\psi + [h - \Omega(I_x + I_y) - K_2 B_\chi B_\psi]\dot{\chi} + K_1 B_\chi^2\chi = 0 \tag{40}$$

where $h \equiv h_r + I_z\Omega$, with h_r the magnitude of the angular momentum of the rotor only. B_χ^2, B_ψ^2, and $B_\chi B_\psi$ are quasiperiodic functions.

In the torque-free case, the motion of the satellite has two natural frequencies, the orbital frequency Ω and the nutational frequency, which can be approximated by $h/\sqrt{(I_xI_y)}$ for a rotor-momentum dominated satellite. In practice, for satellites the orbital mode is usually much slower than the nutational mode. This means that the orbital frequency is much smaller than the nutational frequency, so that

$$\epsilon = \frac{\Omega}{h/\sqrt{I_xI_y}}, \qquad 0 < |\epsilon| \ll 1 \tag{41}$$

Without loss of generality, we assume that at $t = 0$, the satellite is at the ascending node and also $u = 0$, so that $\Omega t = \epsilon\bar{t}$ and $u = \omega_e t = n\epsilon\bar{t}$, where $\bar{t} \equiv [h/\sqrt{(I_xI_y)}]t$ is a nondimensional time and $n \equiv \omega_e/\Omega$ is the ratio of the Earth spin rate to the orbital angular speed of the satellite. Because the magnitude of the geomagnetic field is relatively small, we define

$$\frac{K_1B_o^2}{h^2}\sqrt{I_xI_y} = \epsilon K_1^*, \qquad \frac{K_2B_o^2}{h} = \epsilon K_2^* \tag{42}$$

and assume K_1^* and K_2^* to be of $\mathcal{O}(1)$. Another way to look at this is that the magnetic control torque has a small effect on the nutational frequency of the satellite. This can be understood because the control torque is slowly varying and, hence, it will have a small effect on the frequency of the fast mode of the satellite (nutational mode). By defining $a = \sqrt{(I_x/I_y)}$ and $b = \sqrt{(I_y/I_x)}$ and using the new parameterization, we can write the decoupled equation of motion of the satellite as

$$\chi^{(4)} + \epsilon P_1(\epsilon\bar{t})\chi^{(3)} + P_2(\epsilon\bar{t})\chi^{(2)} + \epsilon P_3(\epsilon\bar{t})\chi^{(1)} + \epsilon^2 P_4(\epsilon\bar{t})\chi = 0 \tag{43}$$

where

$$\begin{aligned}
P_1(\epsilon\bar{t}) &= K_2^*\{(4a+b)S_1(\epsilon\bar{t}) + (4a-b)[S_2(\epsilon\bar{t})\cos 2\epsilon\bar{t} + S_3(\epsilon\bar{t})\sin 2\epsilon\bar{t}]\} + \mathcal{O}(\epsilon) \\
&\equiv p_{11}(\epsilon\bar{t}) + \mathcal{O}(\epsilon) \\
P_2(\epsilon\bar{t}) &= 1 - \epsilon\left\{a + b + 2aK_1^*[S_2(\epsilon\bar{t})\sin 2\epsilon\bar{t} - S_3(\epsilon\bar{t})\cos 2\epsilon\bar{t}]\right\} + \mathcal{O}(\epsilon^2) \\
&\equiv p_{21}(\epsilon\bar{t}) + \epsilon p_{22}(\epsilon\bar{t}) + \mathcal{O}(\epsilon^2) \\
P_3(\epsilon\bar{t}) &= K_1^*[S_1(\epsilon\bar{t}) - S_2(\epsilon\bar{t})\cos 2\epsilon\bar{t} - S_3(\epsilon\bar{t})\sin 2\epsilon\bar{t}] + \mathcal{O}(\epsilon) \\
P_4(\epsilon\bar{t}) &= 1 + nK_1^*[S_4(\epsilon\bar{t}) - S_5(\epsilon\bar{t})\cos 2\epsilon\bar{t} - S_6(\epsilon\bar{t})\sin 2\epsilon\bar{t}] + \mathcal{O}(\epsilon) \\
&\equiv p_{41}(\epsilon\bar{t}) + \mathcal{O}(\epsilon)
\end{aligned} \tag{44}$$

where $(\ )^{(i)}$ denotes the ith derivative with respect to $\bar{t}$ and S_i are slowly periodic functions.

B. Dynamic Analysis Using the GMS Method

By defining a new nondimensional independent variable $\upsilon = \epsilon\bar{t}$ and invoking the GMS technique,[9,10] the variables are extended as $\upsilon \to \{\upsilon_0, \upsilon_1\}$ and $\chi(\upsilon;\epsilon) \to \chi(\upsilon_0, \upsilon_1;\epsilon)$, with $\upsilon_0 = \upsilon$ and $\upsilon_1 = (1/\epsilon)\int k(\upsilon)\,d\upsilon$, where $k(\upsilon)$ is a clock function to be determined. The dominant-order equation by the preceding extension is

$$\mathcal{O}(\epsilon^{-2})\colon\ k^4\frac{\partial^4\chi}{\partial\upsilon_1^4} + k^2\frac{\partial^2\chi}{\partial\upsilon_1^2} = 0 \tag{45}$$

and k has to be selected such that we are able to solve for χ from Eq. (45). For simplicity, we choose $k = 1$. By this choice of k, the time scale υ_1 becomes

$$\upsilon_1 = \upsilon/\epsilon \tag{46}$$

Therefore, the time scale is linear in this case. Note that υ_1 is the fast time scale and υ_0 is the slow time scale.

Using the selected value of k, the solution of Eq. (45) is

$$\chi = A(\upsilon_0)\sin[\upsilon_1 + B(\upsilon_0)] + C(\upsilon_0) \tag{47}$$

The first term in the equation describes the nutational (fast) mode of the satellite, whereas the second term (which is a function of υ_0 only) describes the orbital (slow) mode of the satellite. The two terms in Eq. (47) are two independent solutions of Eq. (45), which are treated separately.

1. *Nutational Mode*

Substitution of the nutational mode $\chi_1 = A(\upsilon_0)\sin[\upsilon_1 + B(\upsilon_0)]$ into the subdominant equation of motion as in the preceding case results in the following amplitude and phase equations:

$$\frac{dA}{d\upsilon_0} - \frac{1}{2}\Big(K_1^*[S_1(\upsilon_0) - S_2(\upsilon_0)\cos 2\upsilon_0 - S_3(\upsilon_0)\sin 2\upsilon_0] - K_2^*\{(4a-b)S_1(\upsilon_0) + (4a-b)[S_2(\upsilon_0)\cos 2\upsilon_0 + S_3(\upsilon_0)\sin 2\upsilon_0]\}\Big)A = 0 \tag{48}$$

and

$$\frac{dB}{d\upsilon_0}+\frac{a+b}{2}+aK_1^*\left[S_2(\upsilon_0)\cos 2\upsilon_0-S_3(\upsilon_0)\sin 2\upsilon_0\right]=0 \quad (49)$$

The solution of Eq. (49) is

$$A(\upsilon_0)=A_0\exp\left\{\tfrac{1}{2}\left[K_1^*-(4a+b)K_2^*\right]U\upsilon_0\right\}\exp\left\{\tfrac{1}{2}\left[K_1^*-(4a+b)K_2^*\right]R_1(\upsilon_0)-\tfrac{1}{2}\left[K_1^*+(4a-b)K_2^*\right]R_2(\upsilon_0)\right\} \quad (50)$$

where U is a positive constant as defined in Eq. (18), whereas $R_1(\upsilon_0)$ and $R_2(\upsilon_0)$ are quasiperiodic functions in υ_0, which depend on the orbit inclination.

Only the first exponential factor in Eq. (50) determines the decay or growth of $A(\upsilon_0)$ and thereby determines the stability of the nutational mode. The second exponential factor contributes only to a periodic variation. Therefore, for an asymptotically stable motion, we must have

$$K_1^*-(4a+b)K_2^* < 0 \quad (51)$$

This is the stability criterion of the nutational mode. By tracing back to the definitions of K_1^* and K_2^*, we can write this stability criterion in terms of K_1 and K_2 as follows:

$$K_1 < \frac{4I_x+I_y}{I_xI_y}hK_2 \quad (52)$$

We see that this criterion depends on the inertia distribution of the satellite and does not depend on the orbit characteristics, except the orbital angular speed, which affects h.

The nutational phase shift is determined by Eq. (49). The solution of this equation is

$$\begin{aligned}B(\upsilon_0)={}&B_0-\frac{1}{2}(a+b)\upsilon_0-aK_1^*\\&\times\left\{\left(\frac{1}{4}\cos^2\gamma\sin^2 i-\frac{1}{8}\sin^2\gamma\sin^2 i\right)\sin 2\upsilon_0\right.\\&-\sin 2\gamma\left[\frac{1}{8(2+n)}\sin 2i-\frac{1}{4(2+n)}\sin i\right]\cos(2+n)\upsilon_0\\&-\sin 2\gamma\left[\frac{1}{8(2-n)}\sin 2i-\frac{1}{4(2-n)}\sin i\right]\cos(2-n)\upsilon_0\\&+\sin^2\gamma\left[\frac{1}{16(1+n)}(1+\cos^2 i)+\frac{1}{8(1+n)}\cos i\right]\\&\times\sin(2+2n)\upsilon_0+\sin^2\gamma\left[\frac{1}{16(1-n)}(1+\cos^2 i)\right.\\&\left.\left.-\frac{1}{8(1-n)}\cos i\right]\sin(2-2n)\upsilon_0\right\}\end{aligned} \quad (53)$$

where B_0 is an arbitrary constant. Note that the phase shift consists of secular and oscillatory terms. The parameters ϵ and n influence the oscillatory terms. This shows the effect of rotation of the Earth on the phase of the nutational mode.

2. *Orbital Mode*

The subdominant-order analysis of the orbital mode leads us to the equation

$$\frac{d^2C}{d\upsilon_0^2}+K_1^*[S_1(\upsilon_0)-S_2(\upsilon_0)\cos 2\upsilon_0-S_3(\upsilon_0)\sin 2\upsilon_0]\frac{dC}{d\upsilon_0}+\left\{1+nK_1^*[S_4(\upsilon_0)-S_5(\upsilon_0)\cos 2\upsilon_0-S_6(\upsilon_0)\sin 2\upsilon_0]\right\}C=0 \quad (54)$$

This shows that the orbital mode is affected by K_1^* but not by K_2^*. Equation (54) is not readily solvable. The limiting values of K_1^* can be obtained from an approximate solution of Eq. (54). The GMS method with nested multiple scaling is utilized to obtain the approximation.

The first limiting case considered is the case where the value of K_1^* is small, that is,

$$K_1^*=\delta\bar{K}_1, \qquad 0<\delta\ll 1 \quad (55)$$

where $\bar{K}_1$ is assumed to be of $\mathcal{O}(1)$. Next the variables in Eq. (54) are extended using $\upsilon_0\to\{\xi_0,\xi_1\}$, $C(\upsilon_0;\delta)\to C(\xi_0,\xi_1;\delta)$, where $\xi_0=\upsilon_0$ and $\xi_1=\delta\int\kappa(\upsilon_0)\,d\upsilon_0$. As before, $\kappa(\upsilon_0)$ is the clock function. By using this extension and continuing the GMS analysis, the following approximation of the orbital mode for small K_1^* is obtained:

$$\begin{aligned}C(\upsilon_0)&=C_0\exp\left(-\tfrac{1}{2}K_1^*U\upsilon_0\right)\exp[G_2(\upsilon_0)]\exp(j\upsilon_0)+cc\\&=C_0\exp\left\{-\tfrac{1}{2}K_1^*U\upsilon_0-\Re[G_2(\upsilon_0)]\right\}\\&\quad\times\sin\{\upsilon_0+\Im[G_2(\upsilon_0)]+C_1\}\end{aligned} \quad (56)$$

where C_0 and C_1 are constants to be determined from the initial conditions and cc are complex conjugates of the preceding term. It can be shown that $G_2(\upsilon_0)$ is only a periodic function around zero equilibrium with a small amplitude. From Eq. (56) the stability of the orbital mode for small K_1^* is determined by the exponential factor. Hence, the stability is determined by $\frac{1}{2}K_1^*U\upsilon_0$, where U is always positive. Hence, for small K_1^*, the orbital mode is asymptotically stable if

$$K_1^* > 0 \leftrightarrow K_1 > 0 \quad (57)$$

To facilitate the stability analysis of the orbital mode for large K_1^* and to implement the nested multiple scales approach, we parameterize

$$K_1^*=(1/\lambda)\bar{K}_1, \qquad 0<\lambda\ll 1 \quad (58)$$

such that $\lambda > \epsilon$ and to be consistent with the definition given by Eq. (42). Equation (54) then can be written as

$$\frac{d^2C}{d\upsilon_0^2}+\lambda^{-1}g_1(\upsilon_0)\frac{dC}{d\upsilon_0}+\left[1+\lambda^{-1}g_2(\upsilon_0)\right]C=0 \quad (59)$$

where g_1 and g_2 are functions defined by comparing Eq. (59) with Eq. (54).

We note that the coefficients containing λ^{-1} are not always large. This is because $g_1(\upsilon_0)$ and $g_2(\upsilon_0)$ are periodic functions that become small and even zero at some υ_0. The characteristic roots of the differential Eq. (59) change back and forth from real to complex values as υ_0 increases. This situation does not happen for the case where K_1^* is small. The point where the two characteristic roots coalesce (on the real axis) is called a turning point. The existence of such points indicates the changes in the topological nature of the solution, for example, from an oscillatory to nonoscillatory behavior. In the present case, there are infinitely many turning points, although all of the turning points lie at about the same place on the real axis. Approximations of the solution will be valid on one side or the other of the turning point. To obtain a uniformly valid approximation of the solution across even a single turning point we need the so-called connection formulas, which are beyond our present scope. Moreover, from a power consideration, a small value of K_1^* is desired. We have already seen that a satisfactory result can be achieved by using small K_1^*. Because of the difficulties mentioned for K_1^* large, to evaluate the stability, numerical simulation is used. The result shows that as long as K_1^* is positive, the stability is guaranteed.

C. Performance Evaluation

From the preceding results, it is also possible to estimate the time constants of the motion for the case where K_1^* is small. The approximate time constant for this case in the nutational mode is

$$T_{c_n}=\frac{2}{\epsilon\left[(4a+b)K_2^*-K_1^*\right](h/\sqrt{I_xI_y})U} \quad (60)$$

and in the orbital mode is

$$T_{c_o} = \frac{2}{\epsilon K_1^*(h/\sqrt{I_x I_y})U} \tag{61}$$

Because U is a positive constant that depends on the inclination of the orbit of the satellite, as in the single-spin case, here also we find that the magnetic control system is more effective for satellites in an orbit at a high angle with respect to the equatorial plane. The true effectiveness of the control system is also determined by the relative position of the geomagnetic dipole with respect to the orbital plane of the satellite. The most effective situation is attained if the geomagnetic dipole lies in the satellite's orbital plane. Some modifications need to be made in the expressions for the approximate time constants to account for this effect. For preliminary analysis purposes, however, an accurate prediction can be achieved using the approximate time constants as given.

The Earth rotation effect mainly causes a slight phase shift in the nutational and orbital modes of the satellite. Thus, for an accurate frequency prediction, this effect should be included.

D. Comparison with Numerical Results

The values of the parameters chosen for this purpose are those of the ITOS satellite in a circular orbit of altitude 1000 km and an inclination of 60 deg. The satellite parameters are $I_x = 155.3$, $I_y = 135.5$, $I_z = 138.9$ kg m^2, and $h = 26.6$ kg m^2. For these parameter values, $\epsilon = 0.0054$.

The stability criteria of the roll/yaw motion of the satellite become

$$K_1^* < 5.2164\, K_2^*, \qquad K_1^* > 0$$

These criteria can be shown to be quite accurate by performing some numerical simulations near the stability boundaries.

Next we select specific values of K_1^* and K_2^* and compare the numerical and GMS results. The specific values selected are $K_1^* = 0.2$ and $K_2^* = 2$ or, equivalently, $K_1 = 1.349 \times 10^6$ A m^2/T and $K_2 = 7.358 \times 10^8$ A m^2 s/T. For the selected numerical values, the approximate frequencies and time constants from the GMS result are $\omega_n = 0.1834$ rad/s and $T_{c_n} = 526$ s (nutational mode) and $\omega_o = 0.000998$ rad/s and $T_{c_o} = 26{,}905$ s (orbital mode). The comparisons of the results are presented in Fig. 3. The nutational and orbital frequencies are shown to be predicted accurately by the GMS method. The time constants of the motion as predicted by GMS method are also accurate. Based on this discussion, we see that, for small values of K_1^*, the approximations obtained by using the GMS approach lead to a good agreement with the exact solution.

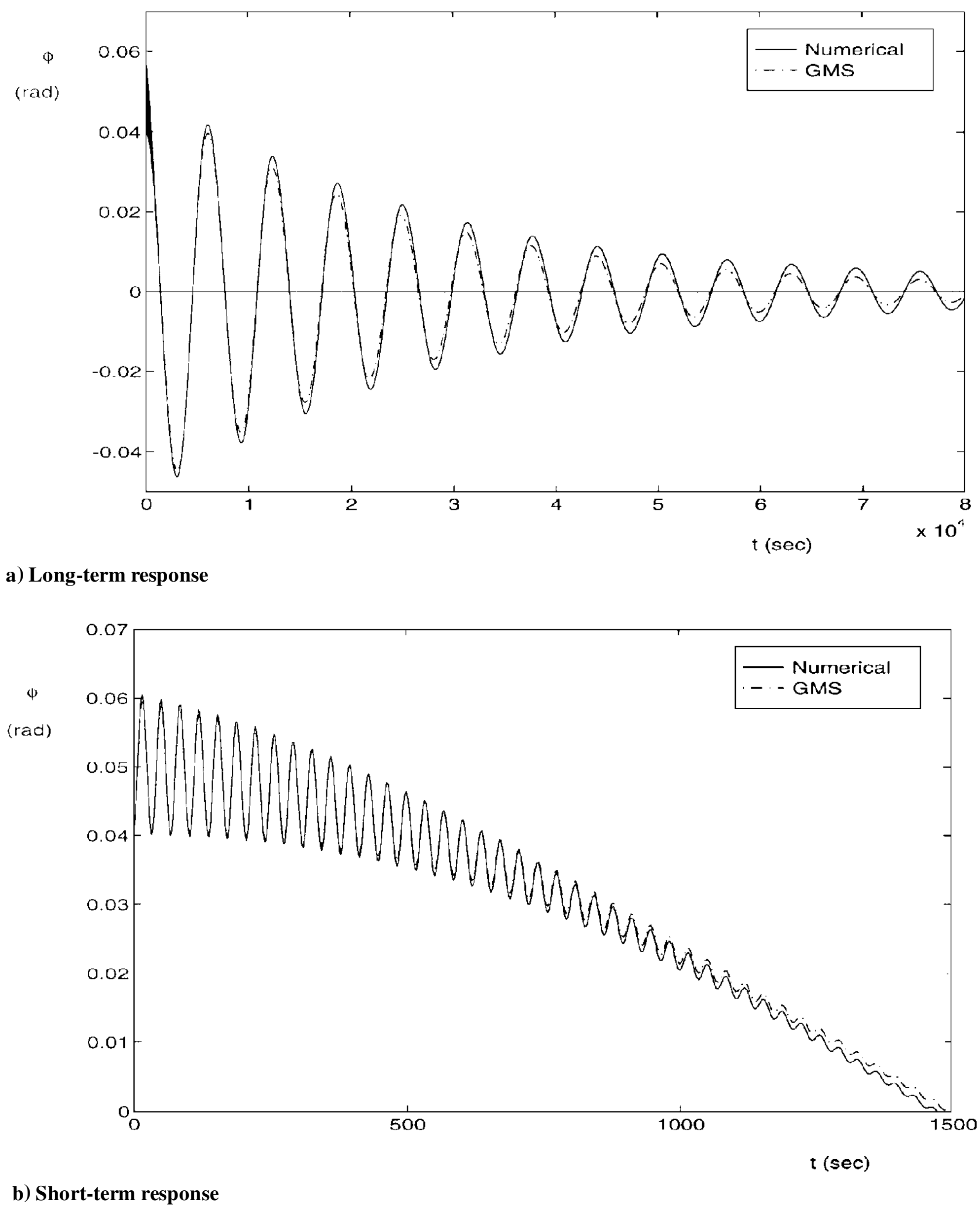

Fig. 3 Comparison of numerical and GMS results for roll motion.

VI. Conclusions

The attitude control of single- and dual-spin satellites using specific control laws that utilize the geomagnetic field has been analyzed. Only satellites in circular Earth orbit are considered. The analysis shows that the control laws considered (with the proper choice of gains) can provide the necessary damping to obtain an asymptotically stable system.

Results in a parametric form were obtained using the GMS method. Each dominant mode of the satellite motion is systematically separated by using proper time scales, leading to insight into the nature of the system dynamics. The time scales used in deriving the results are complex and nonlinear.

A good agreement between the analytical GMS approximations and the numerical integration is also demonstrated. The accuracy can be improved by including higher-order terms in the derivation. Overall, however, for preliminary analysis purposes, the approximations obtained are accurate and lead to considerable insight into the complex nature of the satellite dynamics.

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